Modeling bathymetric uncertainty

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Abstract

Modeling the measurement uncertainty of depth measurement systems, e.g. single-beam and multibeam echosounders, has become common practice since the release of S-44 4th Edition in 1998. Hydrographic Offices have also attempted to model the uncertainty of legacy bathymetric data sets, in order to provide sufficient metadata for users to determine fitness for non-traditional or non-navigation purposes. In addition to estimating the measurement uncertainties associated with depth and position of point soundings, additional uncertainty can be introduced into representative bathymetry models – used in modeling of coastal processes – by various gridding or interpolation methods.

We will begin by briefly reviewing the sources of measurement uncertainty and look at methods for estimating uncertainty in legacy data sets. Next, we look at the uncertainty that is introduced into bathymetry models by various gridding techniques, in essence, estimating the uncertainty of bathymetric values in areas without depth measurements. We will conclude by looking at potential applications of modeled bathymetric uncertainty, including tsunami inundation modeling.

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### Introduction

The initial stimulus for this paper was a request from NGDC to CHS for bathymetry data for the west coast of Canada. The data was requested in order to improve integrated bathymetric-topographic digital elevation models (DEMs) of the west coast of North America for the purpose of tsunami run-up modelling (Figure 1). Tsunamis propagating across the Pacific do not respect political boundaries, so a tsunami originating in Alaska, for example, must necessarily propagate through British Columbia before it reaches other parts of the continental USA. And the better the bathymetry DEM (i.e. the lower the vertical uncertainty), the better will be the veracity of any tsunami run-up modeling in terms of predicting realistic outcomes for inundation.

![Figure 1 - high-resolution bathymetry DEM on the west coast of North America [NGDC, 2011].](image)

An additional stimulus came from one author’s work on the IHO Data Quality Working Group (DQWG), which is presently looking at ways to present uncertainty information about bathymetry to mariners in a more intuitive way. Several ideas on this front came out of an ENC workshop held in January 2011: these ideas are presented again herein.

### Background

It has been shown [e.g. MacEachren et al., 2005] that decisions made with knowledge of data uncertainty are more effective than decisions made without that knowledge, e.g. for bridge risk management when undertaking passage planning. The same could probably be said about algorithmic decisions made using bathymetric uncertainty information when modeling coastal processes.

### Data uncertainty estimation

#### Sources of measurement uncertainty

The basic sources of uncertainty for most of today’s depth and elevation measurement systems, i.e. single-beam and multibeam sonars and bathymetric and topographic lidar, are quite well known. There are sources of uncertainty that contribute only to vertical uncertainty, such as tides, draft and heave, sources of uncertainty that contribute only to horizontal uncertainty, such as horizontal positioning system and heading sensor, and sources of uncertainty that contribute,
through some mapping function to both vertical and horizontal uncertainty, such as range and beam angle uncertainties due to sensor measurement methods and refraction.

Sources of uncertainty can be broken down by:

- Platform:
  - e.g. static draft, vessel speed, changes in draft with loading and speed changes, location of sensors, dynamics of vessel (amount of roll, pitch, heave and yawing);

- Sensor measurements:
  - e.g. sonar, SVP, roll, pitch, heading, heave and positioning (including horizontal datum);

- Environment,
  - e.g. tides (including vertical datum), sound speed structure, sea state;

- Integration:
  - i.e. the time synchronization of all the sensor measurements on a highly dynamic platform; and

- Calibration:
  - e.g. the misalignment angles between the sonar and the motion sensor from a patch test or other calibration method.

There are still other sources of uncertainty in each sonar measurement, such as whether what the sonar detects is the actual seafloor, or due to sonar frequency and the acoustic impedance of the seabed there is some penetration, or due to biological layers or objects, something above the actual seabed is detected. If the sonar beam footprint is larger than the micro-relief of the seabed (e.g. in the case of sandwaves), then some averaged value within the beam footprint may be returned.

When it comes to making inter-comparisons between datasets, temporal changes between the two survey epochs may play a role in expanding the uncertainty of the differences, especially where the seabed is known to be highly mobile or dynamic [e.g. Dorst, 2005]. Precise geo-registration of the datasets is also essential, since any uncertainty in the positions in each will contribute to an inflationary uncertainty in the differences. This uncertainty will be further exaggerated over rugged or steeply sloping seabeds.

As summarized by IHO Standards for Hydrographic Surveys [IHB, 2008], uncertainties associated with the development of the position of an individual beam must include the following:

a) Positioning system errors;
b) Range and beam errors;
c) The error associated with the ray path model (including the sound speed profile), and the beam pointing angle;
d) The error in vessel heading;
e) System pointing errors resulting from transducer misalignment;
f) Sensor location;
g) Vessel motion sensor errors i.e. roll and pitch;
h) Sensor position offset errors; and
i) Time synchronisation / latency.
Contributing factors to the vertical uncertainty include:

a) Vertical datum errors;
b) Vertical positioning system errors;
c) Tidal measurement errors, including co-tidal errors where appropriate;
d) Instrument errors;
e) Sound speed errors;
f) Ellipsoidal / vertical datum separation model errors;
g) Vessel motion errors, i.e. roll, pitch and heave;
h) Vessel draught;
i) Vessel settlement and squat;
j) Seabed slope; and
k) Time synchronisation / latency.

All of these contributing elements can be combined by applying the Law of Propagation of Random Variances, provided all the assumptions that underpin that law are met. This results in estimates of Total Propagated Uncertainty (TPU) for both the vertical (depth/elevation) component (TPU-V) and its corresponding horizontal position (TPU-H). The precise methodology has been well documented for swath (multibeam) systems, e.g. in Hare [1995]. The same methodology could easily be applied to lidar data sets, provided a suitable lidar measurement uncertainty model was available. The single-beam echosounder TPU can be computed as a special case of the multibeam echosounder, where only the nadir beam is considered.

**Estimating uncertainty in legacy data sets**

For legacy data, estimating the uncertainty of position and depth may prove somewhat more challenging. One simple way to obtain a crude estimate is by seeking out the standards that were used to certify the survey at the time it was done. The presumption is that the survey met the standards of the day; therefore all the positions and depths must be at least as good as the specification to which they attempted to adhere. But one must use caution, since assuming a particular standard was followed can lead to incorrect estimates [e.g. Calder, 2006].

Many surveys, in their original form (e.g. fair sheets, field sheets, plans, etc.) may have had good metadata as part of their title blocks or reference notes, or in reports of survey. Oftentimes, information about position accuracy or method of positioning will be available in the metadata. Typical accuracies for many positioning systems and methods have been tabulated [e.g. Hare, 1997] and can be used as a guideline for TPU-H calculation.

The metadata may also include information about the method of depth measurement or the type of echosounder used. These together with any information about how depths were corrected for tides, draft and other biases or scale factors, may lead to a crude estimation of the TPU-V. The method used in S-44 [IHB, 2008] can be applied here, using both fixed \((a)\) and variable \((b)\) contributions to TPU-V as follows:

\[
TPU_V = \pm \sqrt{a^2 + (b \times d)^2}
\]

where \(d\) represents water depth. Note that the coefficients \(a\) and \(b\) would be the quadratic summation of all the contributing fixed and variable uncertainty components respectively.
For analogue survey data, where the data may have become digital through table digitization and may have been transformed from other units into metres and from other datums into e.g. NAD83, these processing errors may contribute to an expansion of the TPU values estimated above. The process by which this expansion occurs also follows the law of propagation of random variances. Methods to compute uncertainty contributions from digitization and processing errors can also be found in the literature, e.g. [ibid.]. The method used to combine any number of uncertainty contributions to position is similar to the equation above:

\[ TPU_H = \pm \sqrt{\sigma_i^2 + \sigma_j^2 + \sigma_k^2 + \ldots} \]

where \( i, j \) and \( k \), etc. are the positioning, digitizing and process errors contributing to the total propagated horizontal uncertainty.

All of the TPU values discussed above can, of course, be scaled to any confidence interval (C.I.) that is needed (often the 95% C.I. is used) using an appropriate expansion factor. For TPU-V, this is 1.96 for normally distributed univariate errors; for TPU-H, a circular distribution is often adopted, and an expansion factor of 2 is used to obtain 95% C.I. estimates.

For a more rigorous approach, one is directed to Calder [2006].

**Model uncertainty estimation**

Models of bathymetry (DEMs) approximate Earth’s surface to some varying degree of accuracy. They are used, especially when integrated with coastal topography in:

- a) the modeling of ocean processes;
- b) coastal and marine spatial planning;
- c) ecosystems and habitat research; and
- d) hazard mitigation and community planning.

They represent, and are derived from, the source elevation data. However, they are typically required to be continuous (i.e., have no gaps) so that ocean phenomena may be modeled using them.

The elevation models are made either of individual cells of uniform size that are in regularly repeating patterns, or of an irregular network of interconnected nodes. Irregular networks have the advantage of more faithfully representing the source elevation data and can be closely spaced where data are dense and more open where data are sparse. Grids with regularly repeating, uniform cell size have the advantage of easy spatial representation and rapid computational use. This last property is especially valuable in operational modeling to support real-time forecasting and warning of tsunamis and hurricane storm-surge.

Each cell in a uniform grid has an assigned elevation value that is expected to be representative of the average elevation of the ground surface within the footprint of the cell; some bathymetry models may use alternative values, such as minimum depth, to support safe navigation. The uncertainty associated with each cell’s elevation value depends therefore upon both the uncertainty of the source data and the gridding technique used to interpolate cell values between soundings (see Figure 2). DEM uncertainties are, in turn, propagated into uncertainty in their applications.
Figure 2 – DEM uncertainty derives from the source data and gridding to interpolate between measured values. Topographic lidar, like multibeam data, is dense with high accuracy, while interpolation is required between sparse, older hydrographic soundings. Earth’s actual surface is presumed to lie somewhere within the DEM uncertainty.

**Contributions from source data (e.g., averaging of multiple data points)**

Where source bathymetry data are present, the uncertainty associated with each sounding value is propagated into the bathymetry model. Where one sounding value corresponds directly to one cell value, the uncertainty of the cell may adopt the uncertainty of that sounding, though differences between the footprints of the sounding measurement and of the cell also contribute. Where multiple soundings in an area are averaged into a single cell value covering them, the uncertainty is derived from either the root-mean square of their individual errors, or from the standard deviation of the source sounding values. Contributing factors include:

a) Uncertainty in source elevation measurement;

b) Footprint differences between source measurement and grid cell;

c) Multiple-value averaging technique used; and

d) Topographic variability within the cell.

**Uncertainty introduced by gridding process**

Where no soundings constrain the depth in an individual cell, or groups of nearby cells, an interpolative gridding technique is required to infer the depth based on surrounding known depths; the modeling of coastal processes typically requires each cell to have an elevation value to prevent instabilities in the models. Common gridding techniques include: spline, kriging, inverse distance weighting (IDW), nearest neighbour, and triangular irregular networks (TINs or tinning) [Maune et al., 2007]. Each technique estimates the depth values using particular constraints, such as a minimum curvature surface for spline, and the uncertainty of the inferred depths will grow with distance from known soundings (Figure 3). Another problem that these techniques introduce is that they may force cell values derived from source soundings away from their average elevation value, adding further uncertainty to those cell values. They may also create gridding artifacts in the model.
There are a number of techniques that can be used to quantify the uncertainty of interpolated elevations, e.g., split-sample, cross-validation, jack-knifing, and boot-strapping [e.g., Erdogan, 2009; Paquet, 2010]. Using a split-sample approach, a percentage of the data is omitted, an interpolation method is applied, and the differences between the interpolated elevations and the original omitted elevations are calculated. This method is often used to assess the stability of various interpolation methods by omitting increasingly greater percentages of the original data and analyzing changes in the uncertainty.

NGDC has written a program that implements the split-sample method (Figure 4). The program intentionally omits a percentage of the xyz points, applies an interpolation method, and calculates the differences between the interpolated values and the omitted elevations. In order to quantify the uncertainty of the interpolation method at every data point, the program repeats this process and aggregates the differences between the original xyz file and the interpolated elevations. The program produces several useful products to be used in assessing the uncertainty of various interpolation methods. The products include a histogram of the differences with statistical measurements, such as the minimum, maximum, mean, root mean squared error (RMSE), and standard deviation. In addition, the program produces a binary grid file of the differences between the original and interpolated elevations.
NOAA’s National Ocean Service (NOS) conducts hydrographic surveys in U.S. waters to support nautical charting. The cleaned and validated bathymetric data from modern surveys are distributed as bathymetric models in the bathymetric attributed grid (BAG) format [ONSWG, 2006], including cell uncertainty (Figure 5). The NOS survey data are available through the NGDC web site: http://www.ngdc.noaa.gov/mgg/bathymetry/hydro.html.

CHS conducts hydrographic surveys in Canadian waters for the same purposes as NOS. These data, in a variety of formats, have been provided to NGDC as part of the tsunami inundation modeling project (discussed in the next section). The data are not publicly available at this time. However, a data discovery portal can be found at: http://www.charts.gc.ca/data-gestion/hydrographic/hydrographic-eng.asp
Applications for DEM with uncertainty

Coastal Inundation Modeling

A primary use of DEMs that integrate bathymetry and topography is the modeling of coastal inundation from either tsunamis or hurricane storm surges [Eakins and Taylor, 2010]. The hydrodynamics of the particular phenomena are modeled upon the DEM, and the location of the maximum inundation line is then used for hazard mitigation planning or operationally during real-time events to help define evacuation areas.

Uncertainty in the cell elevation values directly affects the model hydrodynamics, but they also contribute to horizontal uncertainty of the inundation line. This last piece of information is critical to emergency managers planning for, or responding to, hazard events. The current practice is to assume some additional buffer area beyond the modeled inundation line to use as the basis for decision making. A better practice would be to promulgate DEM uncertainty, along with model uncertainty into a TPU of the inundation line, which would provide more realistic uncertainties on which to base decision making. As yet, neither the storm surge nor tsunami modeling community can accommodate DEM uncertainty, nor track TPU, though this would be a worthy goal.

Bridge risk management

Another application for bathymetry DEM with uncertainty is for voyage planning and risk management on a ship’s bridge. Traditionally, this task has been done using information contained on the paper chart, such as from a source classification diagram (Figure 6), reliability diagram or from notes and symbology on the chart itself. The diagrams, when present, are
always as much smaller scale and contained somewhere within the chart limits, but the information about the quality of the data is never coincident with the data itself. While bathymetry uncertainty is not explicitly stated, it could be crudely implied by experienced mariners and hydrographers from the information given in tabular form.

Figure 6 - example source classification diagram.

More recently, ENCs have been encoded with quality metadata information in the form of zones of confidence, or ZOC. The level of CATZOC, as it is called in the ENC encoding world, can be displayed coincident with the data, allowing decisions to be made with both the data and the uncertainty in context.

Table 1 - Zones of Confidence (ZOC) table (95% C.I.)

<table>
<thead>
<tr>
<th>Zone of Confidence</th>
<th>Horizontal uncertainty</th>
<th>Vertical uncertainty</th>
<th>Seafloor coverage</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1 ***</td>
<td>± 5m + 5% depth</td>
<td>0.5m + 1% depth</td>
<td>Full area search undertaken. Significant seafloor features detected and measured.</td>
</tr>
<tr>
<td></td>
<td>*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A2 ***</td>
<td>± 20m</td>
<td>± 1m + 2% depth</td>
<td>Full area search undertaken. Significant seafloor features detected and measured.</td>
</tr>
<tr>
<td></td>
<td>**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B ***</td>
<td>± 50m</td>
<td>± 1m + 2% depth</td>
<td>Full area search not achieved; uncharted features, hazardous to surface navigation are not expected but may exist.</td>
</tr>
<tr>
<td></td>
<td>*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C ***</td>
<td>± 500m</td>
<td>2m +5% of depth</td>
<td>Full area search not achieved, depth anomalies may be expected.</td>
</tr>
<tr>
<td>D **</td>
<td>Worse than ZOC C</td>
<td>Worse than ZOC C</td>
<td>Full area search not achieved, large depth anomalies may be expected.</td>
</tr>
<tr>
<td>U</td>
<td>Unassessed – The quality of the bathymetric data has yet to be assessed</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In Figure 7, the CATZOCs are represented by the inverted rounded triangle or rounded rectangle symbols, where the number of stars (*) within the triangle/rectangle represents in some way the confidence that a mariner might put in the data (more * are better). Still, this is a discrete representation of the uncertainty information (a continuous variable) and it is not clear how
helpful or intuitive this is for the mariner to make informed decisions about the level of risk-taking by navigating in these areas. The tabular representation of ZOCs is given in Table 1.

Figure 7 - example ENC with CATZOC layer turned on.

With the implementation of new standards for encoding data in ENCs (e.g. BAGs [ONSWG, 2006], S-10x product specification [Ward and Greenslade, 2011], etc.), it should become possible to see and use the depth DEM and its associated uncertainty estimate DEM in the same electronic chart display (e.g. Figure 8).

Figure 8 - Depth (left) and Uncertainty (right) DEM as might be viewed within an Electronic Chart display.

But still more powerful is the combination of the two values into a single layer, with the display customized to the draft of the vessel.
An example might be:
- Subtract 2 times the uncertainty from the charted depth (statistically shoal-biasing it at about the 95% C.I.);
- Apply predicted or real-time tides (biased with their 95% uncertainty if available) to charted values to get real-time shoal-biased depths;
- Apply a vessel draft buffer (the captain’s comfort zone of clearance beneath the keel);
- Colour-code the resultant depths using:
  - Green – where the shoal-biased depths exceed the vessel draft plus draft buffer (a safe-to-go zone);
  - Yellow – where the biased depths exceed the vessel draft, but the buffer is excluded (a cautionary zone); and
  - Red – where depths are not sufficient to navigate the vessel under any circumstance, with the present state of the tides (a no-go zone).

This scenario leads to the traffic-light display shown in Figure 9.

![Figure 9 - example "traffic light" display showing "Go" "No-Go" and "Cautionary" Zones.](image)

**National survey planning**

Knowledge of bathymetry DEM uncertainty can also be used by hydrographic offices (HOs) as a tool for prioritizing work. The CHS developed a risk-classification model [Mortimer, A.R., 2002] for its entire catalogue of charts (some 950) in order to prioritize charting work in an environment of dwindling resources. This model was based, inter alia, on the types and frequency of vessel traffic, the depth of water, the complexity of the areas and on records of accidents and incidents in the area. The report [ibid.] also recommended that CHS apply risk-management approaches to its other planning activities.

One could conceive of using a regional or nation-wide DEM of depths with their associated uncertainty estimates in the development of a national survey plan. Areas where the estimated depth, less its estimated uncertainty is shallower than the draft of expected (or forecast) vessel traffic (with a built-in safety margin) would get the highest priority for resurvey. Of course,
uncertainty estimates would have to also consider the age of the data and the variability of the seafloor when planning a resurvey frequency [e.g. Dorst, L.L., 2005] in order to optimize use of scarce survey resources.

If appropriately modeled in a GIS, this national planning model could be re-run at regular intervals (e.g. annually) or each time major changes occur or are proposed to navigation routes and port facilities.

**Summary and Conclusions**

We have shown the steps involved in estimating bathymetric uncertainty of the source data itself and also those uncertainties due to digitization processes and gridding techniques. In addition, we have shown several applications for bathymetric DEM with associated uncertainty, including bridge risk management, and coastal inundation modeling.

There is certainly the potential for myriad applications of DEM with associated vertical uncertainty estimates. Of the applications examined herein, more work needs to be done on modelling this uncertainty over large areas of coastal and offshore North America, in order to support coastal inundation modeling efforts and for use in risk management applications to support safer marine navigation.

**References**


**Biographies**

Rob Hare is a Manager with Canadian Hydrographic Service. He has worked as a hydrographer and geomatics engineer with CHS since 1982. He is author of numerous papers and reports on hydrographic uncertainty and is Canada’s representative on the IHO working group on Standards for Hydrographic Surveys (S-44) and on the IHO Data Quality Working Group (DQWG). Rob.Hare@dfo-mpo.gc.ca

Barry Eakins is a marine geophysicist and Scientific Lead for the Marine Geology and Geophysics Division and the Coastal Science Team at NGDC, collaborating through the Cooperative Institute for Research in Environmental Sciences (CIRES), University of Colorado at Boulder (CU). His current research efforts focus on the development of high-resolution digital elevation models that integrate coastal bathymetric and topographic data, in support of tsunami inundation modeling for select U.S. coastal regions. He served 6 years in the U.S. Navy, specializing in oceanography and anti-submarine warfare, before obtaining his B.A. in Geology from CU and his Ph.D. in Earth Sciences from Scripps Institution of Oceanography, University of California, San Diego. Barry.Eakins@noaa.gov

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Lisa Taylor is a geophysicist, Leader of the Coastal Science Team, and Director of the International Hydrographic Organization Data Center for Digital Bathymetry (IHODCDB). She is active in the National Tsunami Hazard Mitigation Program and manages NGDC’s effort to build high-resolution digital elevation models (DEMs) for select U.S. coastal regions. These integrated bathymetric-topographic DEMs are used to support the detailed modeling of a variety of natural coastal processes including tsunami inundation, hurricane storm-surge, ocean circulation, sediment transport, and contaminant dispersal. Ms. Taylor is vice chair of the General Bathymetric Chart of the Oceans (GEBCO) Subcommittee on UnderSea Feature Names (SCUFN) and a member of the GEBCO Guiding Committee. She is currently participating in the 18 month NOAA Leadership Competencies Development Program. Lisa.A.Taylor@noaa.gov